Errata in *Optimal Spacecraft Trajectories* by Prussing

July 27, 2018

The most recent version of this Errata page is at

http://prussing.ae.illinois.edu/Errata.OST.pdf

p.5 Sentence continuing at the top is not true in general. It is true for a function of a single variable but not for several variables.

p.10 In Figure 1.1 "Linareacy" should be "L increasing"

p.28 In Eqs. (3.38) and (3.39) the symbol * should be a superscript.
The second line in Eq. (3.38) should be "$= \dot{\mathbf{x}}^*(t_f)dt_f$ + second-order term"

p.28 Add Page 28a (at the end of this Errata) following Page 28.

p.30 Problem 3.1 should say "at the end of the Introduction" (not the Preface).

p.31 Add Problem 3.8: "Show that Eq. (3.46) can be calculated using the function Ω in Eq. (3.47)." (See Page 28a below.)

p.33 In the second line of Example 4.1 the expression for **G(r)** should have no leading minus sign (Eq. 4.6c is correct).

pp.43-44
The denominators in Eqs. 5.3, 5.4, and 5.10 should be scalar magnitudes Δv_o and Δv_f.

p.48 In Fig. 5.3 relabel $\delta\mathbf{r}_o$ as $\delta\mathbf{r}_o = -\Delta\mathbf{v}_o dt_o$.

p.53 In Fig. 5.8 the label at the tip of the $\mathbf{r}_m$ vector should be t_m.

p.61 In Fig. 5.13 left side $\delta\mathbf{v}_f$ should be $\delta\mathbf{r}_f$.
On the top $\mathbf{v}_f^+ - dt_f^+$ should be $\mathbf{v}_f^-\ dt_f$.
On the right side $\mathbf{r}_f^- + d\mathbf{r}_f^-$ should be $\mathbf{r}_f + d\mathbf{r}_f$.
In Eq. (5.80) subscript "F" whould be "f".

p.75 In Fig. 7.1 the trajectory on the right should have label "#1".

p.76 In Fig. 7.2 the ratio b_i/c_i represents the slope of the function m_i/c_i.

p.78 The line after Eq. (7.27) should begin "where i = 1,2 ....".

p.82 Problem 7.1 should state "Is Fig. 7.4 ......".

p.85 In the second line after Eq. (8.4) there should be a space between x * and $f(x)$.
In the fourth line there should be a space between 0 and x *.

p.92 The symbol at the right end of the first line of Eq. (8.38) should be $\Omega_{x(t_f)}$.

p.115 In Eq. (C.24) lower limit on summation sign should be "k=1".
Near the bottom of the page the impulse times should be $t_1 = 0$, $t_2 = 1$, $t_3 = 2$.

p.127 In Eq. (E.15) replace $]^2$ by $)^2$.

p.28a

PAGE 28a (NOT IN TEXTBŌOK): Based on the expression for dJ the NC are

$$\dot{\boldsymbol{\lambda}}^T = -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\partial L}{\partial \mathbf{x}} - \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{3.41}$$

$$\boldsymbol{\lambda}^T(t_f) = \frac{\partial \Phi}{\partial \mathbf{x}(t_f)} \tag{3.42}$$

$$\frac{\partial H}{\partial \mathbf{u}(t)} = \mathbf{0}^T \tag{3.43}$$

$$\text{either} \quad \delta x_k(t_o) = 0 \ \text{ or } \ \lambda_k(t_o) = 0, \quad k = 1, 2, \ldots, n \tag{3.44}$$

$$\boldsymbol{\psi}[\mathbf{x}(t_f), t_f] = \mathbf{0} \tag{3.45}$$

$$\frac{\partial \Phi}{\partial t_f} + L_f + \boldsymbol{\lambda}^T(t_f)\dot{\mathbf{x}}(t_f) = 0 \tag{3.46}$$

Using Eq. (3.42) Eq. (3.46) can be written as $\Omega[\mathbf{x}(t_f), \mathbf{u}(t_f), t_f, \boldsymbol{\nu}] = d\Phi/dt_f + L_f = 0$ but can be more easily *calculated* as

$$\Omega = \partial\Phi/\partial t_f + H_f = 0 \tag{3.47}$$

Note that Ω has the interpretation as $\partial J^*/\partial t_f$, so an optimal value of t_f will satisfy $\Omega = 0$. But if no optimal value of the final time exists, the algebraic sign of Ω indicates how a small change in the final time affects the optimal cost, e.g., if $\Omega < 0$ a small increase in t_f will lower the cost, and vice-versa. That is the case in Example 3.3, where Φ is not an explicit function of the final time, so Ω is equal to the final value of the Hamiltonian.

In most problems, especially if the solution is obtained numerically, an expression for the optimal cost as a function of the final time [Eq. (3.33)] will not be available, but the value of Ω [Eq. (3.34)] will be.

NCs (3.14) and (3.43) are identical but (3.14) was based on the assumption that $\delta\mathbf{u}$ is arbitrary. But if there are terminal constraints this is not true, because only those control variations that generate final states satisfying the terminal constraints are admissible. So this part of the analysis is not rigorous. But a more rigorous treatment in Ref. 3.2 shows that Eq. (3.43) is a correct NC even with terminal constraints.

To summarize, the two new NC are the q terminal constraints in Eq. (3.45) and a scalar NC (3.46) due to t_f being unspecified. So now we have $2n + m + q + 1$ unknowns: $\mathbf{x}, \boldsymbol{\lambda}, \mathbf{u}, \boldsymbol{\nu}, t_f$ and an equal number of NC equations. While it is true that by introducing the Lagrange multiplier $\boldsymbol{\nu}$ we have increased the number of unknowns to solve for, but the vector has a useful interpretation (see Problem 3.4).